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Erratum

Erratum to: "Probability distributions conditioned by the available information: Gamma Distribution and Moments" [Comput. Math. Appl. 52 (2006) 289–304]

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Correction to Eq. (3.3)

$$f(x) = \Gamma(p)\left[\frac{\{\Gamma(p+2n) - q^{-n}\Gamma(p+n)m_{x,n}\} + \{q^{-2n}m_{x,n}\Gamma(p) - q^{-n}\Gamma(p+n)\}x^n}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)}\right] \cdot \frac{x^{p-1}e^{-\frac{x}{q}}}{q^p\Gamma(p)}$$
$$\text{for } m_{x,n} \in \left[\frac{\Gamma(p+n)q^n}{\Gamma(p)}, \frac{\Gamma(p+2n)q^n}{\Gamma(p+n)}\right].$$

Eq. (3.3) can also be rewritten as:

$$f(x) = w_1 \cdot g(x; p, q) + w_2 \cdot g(x; p+n, q)$$

where

$$w_1 = \Gamma(p)\left[\frac{\{\Gamma(p+2n) - q^{-n}\Gamma(p+n)m_{x,n}\}}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)}\right],$$
$$w_2 = \Gamma(p+n)\left[\frac{\{q^{-n}m_{x,n}\Gamma(p) - \Gamma(p+n)\}}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)}\right],$$
$$g(x; p, q) = \frac{x^{p-1}e^{-\frac{x}{q}}}{q^p\Gamma(p)}, \quad x > 0,$$
$$g(x; p+n, q) = \frac{x^{p+n-1}e^{-\frac{x}{q}}}{q^{p+n}\Gamma(p+n)}, \quad x > 0.$$

Correction to Eq. (3.4)

$$0 \le \chi^2_{\min}(f, g) \le \frac{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)}{\Gamma^2(p+n)}.$$

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Correction to Eq. (3.5)

$$\mu_{t,f} = \frac{q^t\Gamma(p+t)\Gamma(p+2n) - m_{x,n}q^{t-n}\Gamma(p+n)\Gamma(p+t)}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)} + \frac{m_{x,n}q^{t-n}\Gamma(p)\Gamma(p+n+t) - q^t\Gamma(p+n)\Gamma(p+n+t)}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)}.$$

Correction to Eq. (3.6)

$$\mu_f = \frac{m_{x,n}nq^{1-n}\Gamma(p)\Gamma(p+n) + qp\Gamma(p)\Gamma(p+2n) - q(p+n)\Gamma^2(p+n)}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)}.$$

Correction to Eq. (3.7)

$$\sigma_f^2 = \frac{\begin{array}{l} q^2\Gamma(p+2)\Gamma(p+2n) - m_{x,n}q^{2-n}\Gamma(p+2)\Gamma(p+n) \\ +q^{2-n}m_{x,n}\Gamma(p)\Gamma(p+n+2) - q^2\Gamma(p+n+2)\Gamma(p+n)\end{array}}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)} - \mu_f^2.$$

Correction to Eq. (3.8)

$$F(x) = \frac{\Gamma(p)\{\Gamma(p+2n) - m_{x,n}q^{-n}\Gamma(p+n)\}}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)} \cdot G(x; p, q) + \frac{\Gamma(p+n)\{m_{x,n}q^{-n}\Gamma(p) - \Gamma(p+n)\}}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)} \cdot G(x; p+n, q).$$

Correction to Eq. (3.10)

$$S(x) = \frac{\Gamma(p)\{\Gamma(p+2n) - m_{x,n}q^{-n}\Gamma(p+n)\}}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)} \cdot [1 - G(x; p, q)] + \frac{\Gamma(p+n)\{m_{x,n}q^{-n}\Gamma(p) - \Gamma(p+n)\}}{\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)} \cdot [1 - G(x; p+n, q)].$$

Correction to Eq. (3.11)

$$h(x) = \frac{\begin{array}{l}\Gamma(p)\{\Gamma(p+2n) - m_{x,n}q^{-n}\Gamma(p+n)\} \cdot g(x; p, q) \\ +\Gamma(p+n)\{m_{x,n}q^{-n}\Gamma(p) - \Gamma(p+n)\} \cdot g(x; p+n, q)\end{array}}{\begin{array}{l}\Gamma(p)\{\Gamma(p+2n) - mq^{-n}\Gamma(p+n)\} \cdot [1 - G(x; p, q)] \\ +\Gamma(p+n)\{m_{x,n}q^{-n}\Gamma(p) - \Gamma(p+n)\} \cdot [1 - G(x; p+n, q)]\end{array}}$$

Correction to Corollary 3.1. For $m = q^n\Gamma(p+n)/\Gamma(p)$, the …

Correction to Corollary 3.2. For $m = q^n\Gamma(p+2n)/\Gamma(p+n)$, the probability …

Correction to Eq. (3.12). [The existing function is not a PDF]

$$f(x) = \frac{1}{q^{p+n}\Gamma(p+n)} \cdot x^{p+n-1}e^{-\frac{x}{q}}, \quad x > 0.$$

Corollary 3.3. For $m \in \left(\frac{q^n\Gamma(p+n)}{\Gamma(p)}, \frac{q^n\Gamma(p+2n)}{\Gamma(p+n)}\right)$, we have $f(x)$ as expressed earlier is a weighted mixture of two gamma densities with the pre-described weights.

For example, if

$$m = \frac{1}{2}\left(\frac{q^n\Gamma(p+n)}{\Gamma(p)} + \frac{q^n\Gamma(p+2n)}{\Gamma(p+n)}\right),$$

then

Correction to Eq. (3.13)

$$f(x) = \frac{1}{2}g(x; p, q) + \frac{1}{2}g(x; p+n, q).$$

Correction to Eq. (3.14)

$$\chi^2_{\min}(f, g) = \frac{\left(\Gamma(p)\Gamma(p+2n) - \Gamma^2(p+n)\right)^2}{4\Gamma^2(p+n)}.$$

3.1 Case $n = 1$, i.e. given arithmetic Mean $E(X)$

Correction to Eq. (3.17)

$$f(x) = \left[\left(\frac{q(p+1)-m}{q}\right) + \left(\frac{m-pq}{q^2 p}\right)\cdot x\right]\cdot\frac{x^{p-1}\mathrm{e}^{-x/q}}{q^p\Gamma(p)}$$

for $m \in [qp, q(p+1)]$.

Also, Eq. (3.17) can be rewritten in the form of a weighted mixture of two gamma probability density functions with parameters (p, q) and $(p+1, q)$ and with respective weights $[\{q(p+1)-m\}/q]$ and $[\{m-pq\}/q]$.

$$\begin{aligned}
&f(x) = w_1 g(x; p, q) + w_2 g(x; p+1, q),\\
&w_1 = \frac{q(p+1)-m}{q},\\
&w_2 = \frac{m-pq}{q},\\
&g(x; p, q) = \frac{x^{p-1}\mathrm{e}^{-x/q}}{q^p\Gamma(p)}, \quad x > 0,\\
&g(x; p+1, q) = \frac{x^{p}\mathrm{e}^{-x/q}}{q^{p+1}\Gamma(p+1)}, \quad x > 0.
\end{aligned}$$

Correction to Eq. (3.18)

$$\chi^2_{\min}(f, g) = \frac{(m-qp)^2}{q^2 p}.$$

Correction to Eq. (3.19)

$$0 \le \chi^2_{\min}(f, g) \le \frac{1}{p}.$$

Correction to Eq. (3.20)

$$\mu_{t,f} = \frac{q^{t-1}\Gamma(p+t)}{\Gamma(p+1)}[qp(1-t)+mt].$$

The variance

$$\sigma_f^2 = q(p+1)(2m-pq) - m^2.$$

Correction to Eq. (3.21)

$$F(x) = \left(\frac{q(p+1)-m}{q}\right)\cdot G(x; p, q) + \left(\frac{m-pq}{q}\right)\cdot G(x; p+1, q).$$

Correction to Eq. (3.22)

$$S(x) = \left(\frac{q(p+1)-m}{q}\right)\cdot[1 - G(x; p, q)] + \left(\frac{m-pq}{q}\right)\cdot[1 - G(x; p+1, q)].$$

Correction to Eq. (3.23)

$$h(x) = \frac{(q(p+1)-m)\cdot g(x; p, q) + (m-pq)\cdot g(x; p+1, q)}{(q(p+1)-m)\cdot[1-G(x; p, q)] + (m-pq)\cdot[1-G(x; p+1, q)]}.$$

Correction to Corollary 3.5. For $m = q(p+1)$, the …

Correction to Corollary 3.6. For $m \in (qp, q(p+1))$, the …

Correction to Eq. (3.26)

$$\chi^2_{\min}(f, g) = \frac{1}{4p}.$$

Correction to Table 1

Table 1
The minimum χ^2-divergence probability distributions given: gamma distribution and arithmetic mean.

p, q	Mean	$f(x)$	$\chi^2_{\min}$	Figure
1, 0.5	0.5	$g(x) = 2\exp[-2x], x > 0$	0	1a
	1	$g_1(x) = 4x\exp[-2x], x > 0$	1	
	0.75	$h(x) = 0.5g(x) + 0.5g_1(x), x > 0$	0.25	
2, 1	2	$g(x) = x\exp[-x], x > 0$	0	1b
	3	$g_1(x) = 0.5x^2\exp[-x], x > 0$	0.5	
	2.5	$h(x) = 0.5g(x) + 0.5g_1(x), x > 0$	0.125	
2, 3	6	$g(x) = 9^{-1}x\exp[-x/3], x > 0$	0	1c
	9	$g_1(x) = 54^{-1}x^2\exp[-x/3], x > 0$	0.5	
	7.5	$h(x) = 0.5g(x) + 0.5g_1(x), x > 0$	0.125	
3, 2	6	$g(x) = 16^{-1}x^2\exp[-x/2], x > 0$	0	1d
	8	$g_1(x) = 96^{-1}x^3\exp[-x/2], x > 0$	0.333	
	7	$h(x) = 0.5g(x) + 0.5g_1(x), x > 0$	0.083	

3.2 Case $n = 2$, i.e. given second moment $E(X^2) = b$

Correction to Eq. (3.29)

$$f(x) = \frac{1}{2q^2(2p+3)}\left[\{q^2(p+2)(p+3) - b\} + \frac{b - q^2p(p+1)}{q^2p(p+1)} \cdot x^2\right] \cdot \frac{x^{p-1}e^{-x/q}}{q^p\Gamma(p)}$$

for

$$b \in \left[q^2p(p+1), q^2(p+2)(p+3)\right].$$

Eq. (3.29) can be rewritten in the form of a weighted mixture of two gamma densities with parameters (p, q) and $(p+2, q)$.

$$f(x) = w_1 g(x; p, q) + w_2 g(x; p+2, q),$$

where

$$w_1 = \frac{q^2(p+2)(p+3) - b}{2q^2(2p+3)},$$

$$w_2 = \frac{b - q^2p(p+1)}{2q^2(2p+3)},$$

$$g(x; p, q) = \frac{x^{p-1}e^{-x/q}}{q^p\Gamma(p)}, \quad x > 0,$$

$$g(x; p+2, q) = \frac{x^{p+1}e^{-x/q}}{q^{p+2}\Gamma(p+2)}, \quad x > 0.$$

Correction to Eq. (3.30)

$$0 \le \chi^2_{\min}(f, g) \le \frac{(m - q^2p(p+1))^2}{2q^4p(p+1)(2p+3)}.$$

Correction to Eq. (3.31)

$$\mu_{t,f} = \left[\{q^2(p+2)(p+3) - b\} + \frac{\{b - q^2p(p+1)\}}{p(p+1)}(p+t)(p+t+1)\right] \cdot \frac{q^{t-2}\Gamma(p+t)}{2(2p+3)\Gamma(p)}.$$

Correction to the Mean

$$\mu_f = \frac{q^2p(p+2) + b}{q(2p+3)}.$$

Correction to the Variance

$$\sigma_f^2 = b - \left(\frac{q^2p(p+2) + b}{q(2p+3)}\right)^2.$$

Correction to Eq. (3.32)

$$F(x)=\frac{[q^2(p+2)(p+3)-b]G(x;p,q)+[b-q^2p(p+1)]G(x;p+2,q)}{2q^2(2p+3)}.$$

Correction to Eq. (3.33)

$$S(x)=\frac{[q^2(p+2)(p+3)-b]\cdot[1-G(x;p,q)]+[b-q^2p(p+1)]\cdot[1-G(x;p+2,q)]}{2q^2(2p+3)}.$$

Correction to Eq. (3.34)

$$h(x)=\frac{[q^2(p+2)(p+3)-b]\cdot g(x;p,q)+[b-q^2p(p+1)]\cdot g(x;p+2,q)}{[q^2(p+2)(p+3)-b]\cdot[1-G(x;p,q)]+[b-q^2p(p+1)]\cdot[1-G(x;p+2,q)]}.$$

Correction to Corollary 3.7. For $b=q^2p(p+1)$, the …

Correction to Corollary 3.8. For $b=q^2(p+2)(p+3)$, the …

Correction to Eq. (3.35)

$$f(x)=g(x;p+2,q)=\frac{1}{q^{p+2}\Gamma(p+2)}x^{p+1}\mathrm{e}^{-x/q},\quad x>0.$$

Correction to Corollary 3.9. For $b\in\left(q^2p(p+1),q^2(p+2)(p+3)\right)$, the …

Correction to Eq. (3.36)

$$f(x)=\frac{1}{2}\left[1+\frac{1}{q^2p(p+1)}x^2\right]\frac{x^{p-1}\mathrm{e}^{-x/q}}{q^p\Gamma(p)}.$$

Or, simply

$$f(x)=\frac{1}{2}g(x;p,q)+\frac{1}{2}g(x;p+2,q).$$

Correction to Table 2

Table 2
The minimum χ^2-divergence probability distributions given: gamma distribution and second moment.

p, q	$E(X^2)$	$f(x)$	$\chi^2_{\min}$	Figure
	0.5	$g(x)=2\exp[-2x]$, $x>0$	0	
1, 0.5	3	$g_1(x)=4x^2\exp[-2x]$, $x>0$	5	2a
	1.75	$h(x)=0.5g(x)+0.5g_1(x)$, $x>0$	4	
	6	$g(x)=x\exp[-x]$, $x>0$	0	
2, 1	20	$g_1(x)=6^{-1}x^3\exp[-x]$, $x>0$	2.833	2b
	13	$h(x)=0.5g(x)+0.5g_1(x)$, $x>0$	0.583	
	54	$g(x)=9^{-1}x\exp[-x/3]$, $x>0$	0	
2, 3	180	$g_1(x)=(486)^{-1}x^3\exp[-x/3]$, $x>0$	2.333	2c
	117	$h(x)=0.5g(x)+0.5g_1(x)$, $x>0$	0.583	
	48	$g(x)=16^{-1}x^2\exp[-x/2]$, $x>0$	0	
3, 2	120	$g_1(x)=(768)^{-1}x^4\exp[-x/2]$, $x>0$	1.5	2d
	84	$h(x)=0.5g(x)+0.5g_1(x)$, $x>0$	0.375	

4. Given a gamma distribution and information on logarithmic mean, $E(\ln X)$

Correction to Eq. (4.8)

$$\mu_{t,f}=\frac{1}{\sigma^2_{\ln X,g}}\left[(m_{\ln^2X,g}-Gm_{\ln X,g})\cdot\frac{q^t\Gamma(p+t)}{\Gamma(p)}+(G-m_{\ln X,g})m_{X^t\ln X,g}\right].$$

Correction to the Variance

$$\sigma^2_f=\frac{1}{\sigma^2_{\ln X,g}}\left[(m_{\ln^2X,g}-Gm_{\ln X,g})\cdot q^2p(p+1)+(G-m_{\ln X,g})m_{X^2\ln X,g}\right]-\mu^2_f.$$

Correction to Eq. (4.10)

$$F(x) = \frac{(m_{\ln^2 X,g} - G m_{\ln X,g}) \cdot G(x; p, q) + (G - m_{\ln X,g}) m_{\ln X,g} \cdot G_{\ln X}(x; p, q)}{\sigma^2_{\ln X,g}}.$$

Correction to Eq. (4.11)

$$G_{\ln X}(x; p, q) = \int_0^x \frac{1}{m_{\ln X,g} q^p \Gamma(p)} \cdot (\ln w) w^{p-1} \mathrm{e}^{-w/q} \mathrm{d}w, \quad x > 0.$$

Correction to Eq. (4.12)

$$S(x) = \frac{(m_{\ln^2 X,g} - G m_{\ln X,g}) \cdot [1 - G(x; p, q)] + (G - m_{\ln X,g}) m_{\ln X,g} \cdot [1 - G_{\ln X}(x; p, q)]}{\sigma^2_{\ln X,g}}.$$

Correction to Eq. (4.13)

$$h(x) = \frac{(m_{\ln^2 X,g} - G m_{\ln X,g}) \cdot g(x; p, q) + (G - m_{\ln X,g}) m_{\ln X,g} \cdot g_{\ln X}(x; p, q)}{(m_{\ln^2 X,g} - G m_{\ln X,g}) \cdot [1 - G(x; p, q)] + (G - m_{\ln X,g}) m_{\ln X,g} \cdot [1 - G_{\ln X}(x; p, q)]}.$$

Correction to Eq. (4.14)

$$f(x) = g_{\ln X}(x; p, q) = \frac{1}{m_{\ln X,g} q^p \Gamma(p)} \cdot (\ln x) x^{p-1} \mathrm{e}^{-x/q}, \quad x > 0.$$

5. Given a gamma distribution and information on $E(X)$ and $V(X)$

Correction to Eq. (5.3)

$$m_t = q^t \frac{\Gamma(p+t)}{\Gamma(p)}.$$

Correction to Eq. (5.5)

$$\mu_{t,f} = \frac{q^t \Gamma(p+t)}{\Gamma(p)} [\alpha_0 + q(p+t)\{\alpha_1 + q(p+t+1)\alpha_2\}].$$

Correction to Eq. (5.6)

$$\mu_f = qp [\alpha_0 + q(p+1)\{\alpha_1 + q(p+2)\alpha_2\}].$$

Correction to Eq. (5.7)

$$\sigma_f^2 = q^2 p(p+1) [\alpha_0 + q(p+2)\{\alpha_1 + q(p+3)\alpha_2\}] - \mu_f^2.$$

Correction to Table 3

Table 3
The minimum χ^2-divergence probability distributions given: gamma distribution and first and second moments.

p, q	$E(X), E(X^2)$	f(x)	$\chi^2_{\min}$	Figure
1, 0.5	0.5, 0.5	$g(x) = 2\exp[-2x], x > 0$	0	3a
	1, 1.5	$g_1(x) = 4x\exp[-2x], x > 0$	1	
	1.5, 3	$g_2(x) = 4x^2\exp[-2x], x > 0$	5	
	1, 1.75	$h(x) = 0.5g(x) + 0.5g_2(x), x > 0$	1.25	
3, 2	6, 48	$g(x) = (16)^{-1}x^2\exp[-x/2], x > 0$	0	3b
	8, 80	$g_1(x) = (96)^{-1}x^3\exp[-x/2], x > 0$	0.333	
	10, 120	$g_2(x) = (768)^{-1}x^4\exp[-x/2], x > 0$	1.5	
	9, 100	$h(x) = 0.5g_1(x) + 0.5g_2(x), x > 0$	0.792	

Correction to Fig. 1

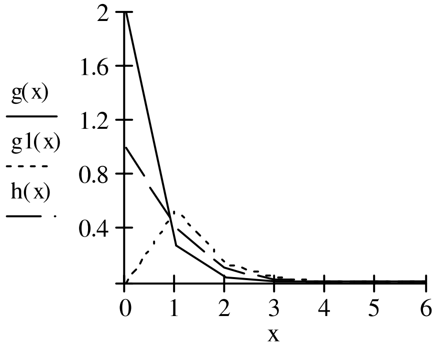

Fig. 1a. X has Gamma $(1, 0.5)$, $m = E(X) \in [0.5, 1]$.

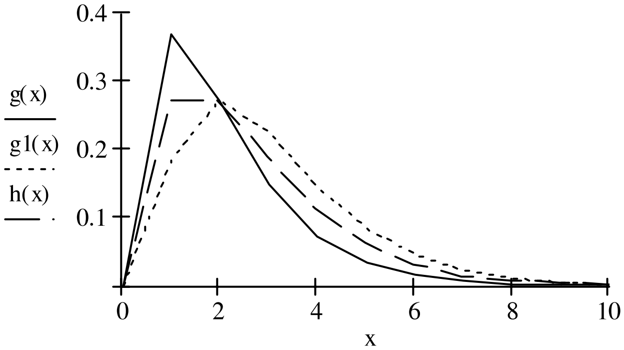

Fig. 1b. X has Gamma $(2, 1)$, $m = E(X) \in [2, 3]$.

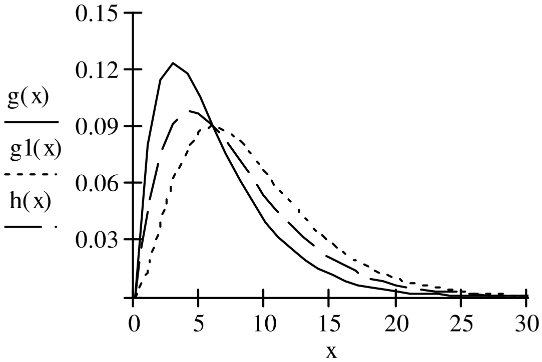

Fig. 1c. X has Gamma $(2, 3)$, $m = E(x) \in [6, 9]$.

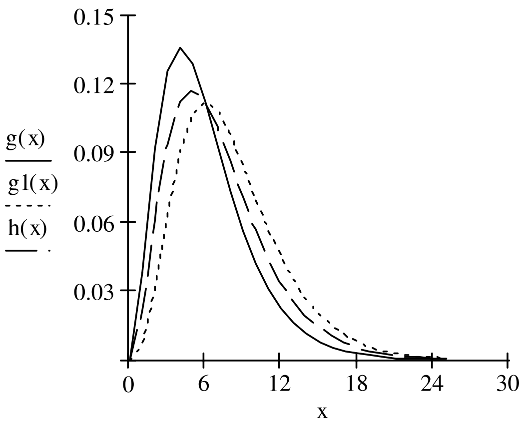

Fig. 1d. X has Gamma $(3, 2)$, $m = E(x) \in [6, 8]$.

Correction to Fig. 2

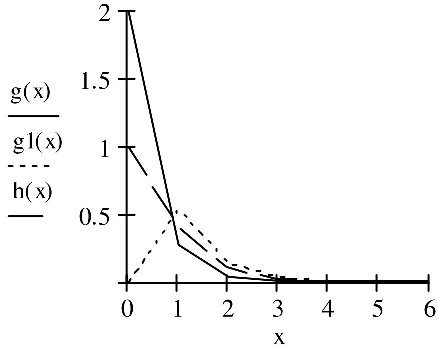

Fig. 2a. X has Gamma $(1, 0.5)$, $b = E(X^2) \in [0.5, 3]$.

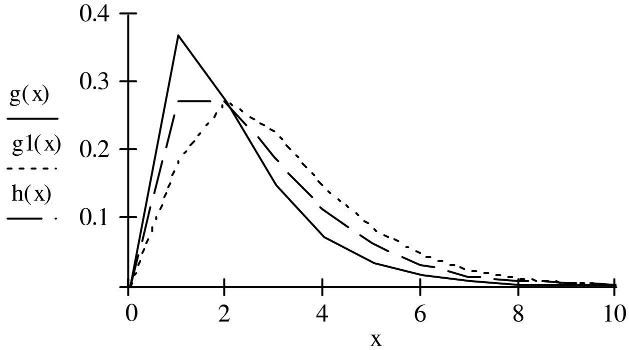

Fig. 2b. X has Gamma $(2, 1)$, $b = E(X^2) \in [6, 20]$.

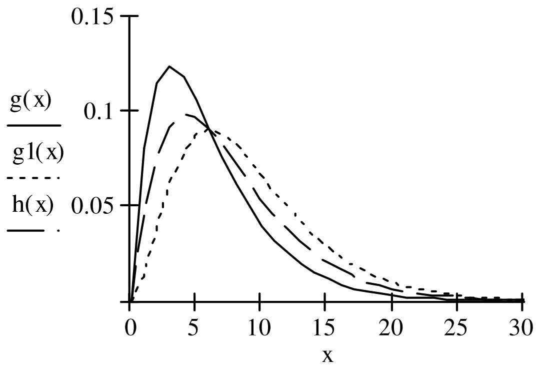

Fig. 2c. X has Gamma $(2, 3)$, $b = E(X^2) \in [54, 180]$.

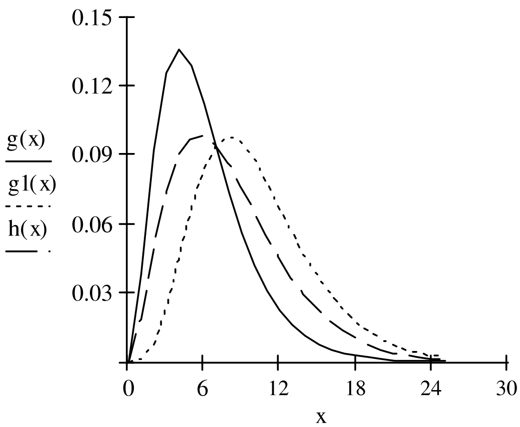

Fig. 2d. X has Gamma $(3, 2)$, $b = E(X^2) \in [48, 120]$.

Correction to Fig. 3

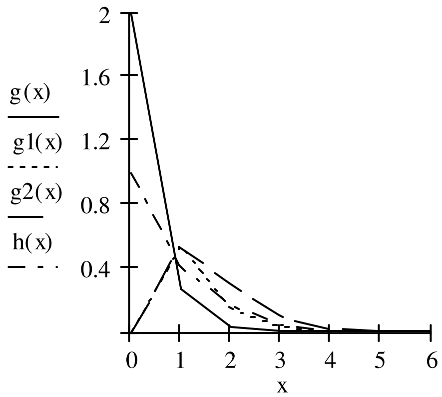

Fig. 3a. X has Gamma $(1, 0.5)$, $a \in [0.5, 1.5]$, $b \in [0.5, 3]$.

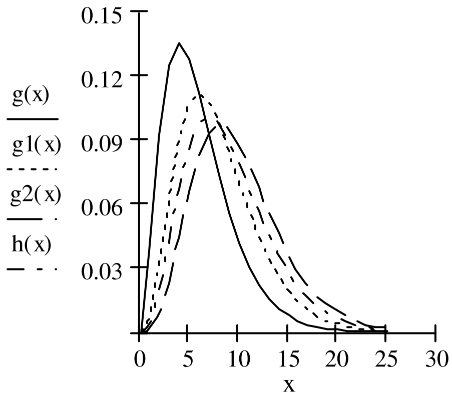

Fig. 3b. X has Gamma $(3, 2)$, $a \in [6, 10]$, $b \in [48, 120]$.